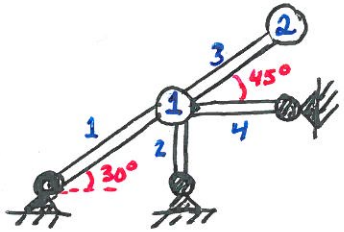
EX: For the truss drawn below, split the force into components

- f_b balanced by tension/comp.
- f_m causing motion of system



$f_1^x = 1$ $f_2^x = 1$

$f_1^y = -1$ $f_2^y = 0$

First we will write the elongation matrix

$$A = \begin{array}{cccccl} & \text{Joint 1} & & \text{Joint 2} & & \\ & x & y & x & y & \\ & \downarrow & \downarrow & \downarrow & \downarrow & \\ \left[\right. & \sqrt{3}/2 & 1/2 & 0 & 0 & \left.\right] \to \text{bar 1} \\ & 0 & 1 & 0 & 0 & \to \text{bar 2} \\ & -\sqrt{2}/2 & -\sqrt{2}/2 & \sqrt{2}/2 & \sqrt{2}/2 & \to \text{bar 3} \\ & -1 & 0 & 0 & 0 & \to \text{bar 4} \end{array}$$

Reduce A to get nullspace, etc.

$$A \longrightarrow \begin{array}{cccccc} u_1^x & u_1^y & u_2^x & u_2^y & & \\ \boxed{\sqrt{3}/2} & 1/2 & 0 & 0 & \to = 0 & \sqrt{3}/2\, u_1^x = -1/2\, u_1^y \\ 0 & \boxed{1} & 0 & 0 & \to = 0 & u_1^y = 0 \\ 0 & 0 & \boxed{\sqrt{2}/2} & \sqrt{2}/2 & \to = 0 & \sqrt{2}/2\, u_2^x = -\sqrt{2}/2\, u_2^y \\ 0 & 0 & 0 & 0 & \to = 0 & \\ \uparrow & \uparrow & \uparrow & \uparrow & & \\ \text{pivots} & & & \text{free} & & \end{array}$$

$$\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

Nullspace basis

Nullspace basis: $\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$ ← Possible Motions.

Rowspace basis: $\left\{ \begin{array}{l} [1\ 0\ 0\ 0], \\ [0\ 1\ 0\ 0], \\ [0\ 0\ \sqrt{2}/2\ \sqrt{2}/2] \end{array} \right\}$ simplify $\times\sqrt{2}$ $\rightsquigarrow [0\ 0\ 1\ 1]$

Transpose

A^T has column space basis: $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$

Balanced Forces.

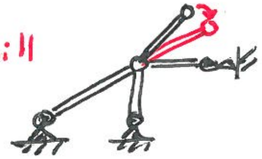
Note: These are ⊥

- f_m = Projection of f onto nullspace of A
  - component in direction of motion
- f_b = Projection of f onto column space of A^T
  - component ⊥ to motion
  - in direction of bars

$$f_m = \frac{\begin{bmatrix}0\\0\\-1\\1\end{bmatrix}\cdot\begin{bmatrix}1\\-1\\1\\0\end{bmatrix}}{\begin{bmatrix}0\\0\\-1\\1\end{bmatrix}\cdot\begin{bmatrix}0\\0\\-1\\1\end{bmatrix}}\begin{bmatrix}0\\0\\-1\\1\end{bmatrix} = -\frac{1}{2}\begin{bmatrix}0\\0\\-1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\1/2\\-1/2\end{bmatrix}$$

joint 2 will rotate:

$\frac{1}{2}\begin{bmatrix}1\\-1\end{bmatrix}$ ← right ← down



Two ways to compute f_b

Method 1. Since we know f_m, use

$$f_b = f - f_m$$

$$f_b = \begin{bmatrix}1\\-1\\1\\0\end{bmatrix} - \begin{bmatrix}0\\0\\1/2\\-1/2\end{bmatrix} = \begin{bmatrix}1\\-1\\1/2\\1/2\end{bmatrix}$$

Note: Sometimes finding f_m will not be so easy — usually nullspace has more than one basis vector... It is important to be able to calculate f_b without using f_m!

Method 2. Project f onto colspace A^T

$f_b = C\hat{x}$ where $\hat{x}$ is approx. solution to $f = Cx$

$\uparrow$ (basis vectors for colspace A^T)

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&1\end{bmatrix}x = \begin{bmatrix}1\\-1\\1\\0\end{bmatrix}$$

$$\Downarrow$$

Normal Eqn $\begin{bmatrix}1&0&0\\0&1&0\\0&0&2\end{bmatrix}\hat{x} = \begin{bmatrix}1\\-1\\1\end{bmatrix}$

$$\hat{x} = \begin{bmatrix}1\\-1\\1/2\end{bmatrix}$$

$$f_b = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&1\end{bmatrix}\begin{bmatrix}1\\-1\\1/2\end{bmatrix} = \begin{bmatrix}1\\-1\\1/2\\1/2\end{bmatrix}$$